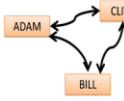
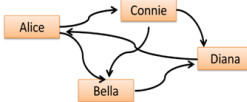
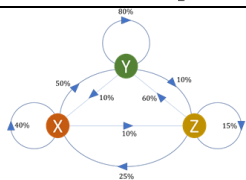


Concept	Definition/ Explanation	Example
Matrix	A rectangular array of numbers arranged in rows and columns	$\begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 3 \\ 0 & 1 & 3 & 0 \end{bmatrix}$
Element	The numbers in a matrix. a_{31} is the element in row 3 and column 1 a_{ij} is the element in row i and column j	$A = \begin{bmatrix} 2 & 3 \\ 1 & 8 \\ 5 & 0 \\ 4 & 6 \end{bmatrix}$ $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix}$
Order of a matrix	The size of the matrix is called its 'order' Number of rows is always written first.	$\begin{bmatrix} 10 & 6 & 2 \\ 7 & 9 & 5 \end{bmatrix}$ This matrix is of the order 2×3
Row Matrix	A matrix with only 1 row ($1 \times n$)	$[5 \quad -1 \quad 3]$
Column Matrix	A matrix with only 1 column ($n \times 1$)	$\begin{bmatrix} 9 \\ 3 \\ 4 \end{bmatrix}$
Square Matrix	Have the same number of rows and columns ($n \times n$)	$\begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 3 \\ 0 & 1 & 3 & 0 \end{bmatrix}$
Diagonal Matrix	A square matrix with non-zero elements in the leading diagonal only	$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
Identity Matrix	A diagonal matrix that has only 1s in the leading diagonal. (Must be a square matrix)	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Transpose of a Matrix	To transpose a matrix, swap its rows and columns. (CAS: use the superscript T to transpose matrix on CAS)	$A = \begin{bmatrix} 2 & 3 & 6 \\ 8 & 5 & 1 \end{bmatrix}$ $A^T = \begin{bmatrix} 2 & 8 \\ 3 & 5 \\ 6 & 1 \end{bmatrix}$
Symmetric Matrix	The matrix and its transpose is the same.	$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$
Upper Triangular Matrix	Square matrix where all elements below the diagonal are 0	$\begin{bmatrix} 3 & 8 & 1 & 9 \\ 0 & 7 & 7 & 3 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$
Lower Triangular Matrix	Square matrix where all elements above the diagonal are 0	$\begin{bmatrix} 8 & 0 & 0 \\ 5 & 2 & 0 \\ 5 & 8 & 1 \end{bmatrix}$
Matrix Addition and Subtraction	Can only add or subtract matrices that are of the same order. Each element is added/subtracted from their corresponding element.	$M = \begin{bmatrix} 4 & 8 & 1 \\ 0 & 7 & -3 \end{bmatrix}$ $N = \begin{bmatrix} -1 & 5 & 9 \\ 4 & 2 & 1 \end{bmatrix}$ $M - N = \begin{bmatrix} 4 - (-1) & 8 - 5 & 1 - 9 \\ 0 - 4 & 7 - 2 & -3 - 1 \end{bmatrix}$ $= \begin{bmatrix} 5 & 3 & -8 \\ -4 & 5 & -4 \end{bmatrix}$
Scalar Multiplication	Each element is multiplied.	$0.5 \begin{bmatrix} 4 & 5 & 2 \\ 7 & 8 & 1 \\ 4 & 1 & 12 \end{bmatrix}$ $= \begin{bmatrix} 4 \times 0.5 & 5 \times 0.5 & 2 \times 0.5 \\ 7 \times 0.5 & 8 \times 0.5 & 1 \times 0.5 \\ 4 \times 0.5 & 1 \times 0.5 & 12 \times 0.5 \end{bmatrix} = \begin{bmatrix} 2 & 2.5 & 1 \\ 3.5 & 4 & 0.5 \\ 2 & 0.5 & 6 \end{bmatrix}$

Concept	Definition / Explanation	Example
Matrix Multiplication	The number of columns in the first matrix must be equal to the number of rows in the second. Matrix multiplication is not commutative $AB \neq BA$	$A = \begin{bmatrix} 6 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ $A \times B = \begin{bmatrix} 6 \times 3 + 5 \times 1 & 6 \times 4 + 5 \times 2 \end{bmatrix}$ $A \times B = \begin{bmatrix} 23 & 34 \end{bmatrix}$
Matrix product defined	For a matrix product to be defined, you should be able to multiply the matrices.	$A = \begin{bmatrix} 6 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ AB is defined as the number of columns of A = the number of rows in B . BA is not defined as the number of columns of $B \neq$ the number of rows in A .
Powers of a matrix	Only square matrices can be raised to a power	$A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$ $A^3 = A \times A \times A$
Binary Matrix	A matrix that contains only 0s and 1s	$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$
Permutation Matrix	A square binary matrix that has only one 1 in each row and each column.	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Rearranging a Matrix	A permutation matrix can be used to re-arrange a matrix.	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} T \\ O \\ P \end{bmatrix} = \begin{bmatrix} O \\ T \\ P \end{bmatrix}$
Identity Matrix	Is a square binary matrix that has 1s only in the leading diagonal. Special feature: any matrix multiplied by the identity matrix is itself.	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} O \\ T \\ P \end{bmatrix} = \begin{bmatrix} O \\ T \\ P \end{bmatrix}$
Inverse Matrix	If you multiply a matrix with its inverse you get the identity matrix.	$A \times A^{-1} = I$
Inverse of a 2×2 matrix	Swap the elements of the leading diagonal. Swap signs of the other elements. Multiply by $1/\text{determinant}$	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
Determinant of a 2×2 matrix	To find the determinant on CAS use " $\text{Det}(A)$ "	$A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ $\det(A) = 2 \times 4 - 3 \times 1 = 5$
Singular Matrix	A matrix that has a determinant of 0 is called a singular matrix. You cannot find the inverse of a singular matrix.	$B = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$ $\det(B) = 12 - 12 = 0$
Solving Matrix equations	Use inverse matrices to solve an equation	$AX = B$ $A^{-1}AX = A^{-1}B$ Pre-multiply by A^{-1} on both sides $IX = A^{-1}B$ Since $A^{-1}A = I$ $X = A^{-1}B$ Since $IX = X$
Setting up simultaneous equations in matrix form	Setup simultaneous equations using matrices. Then use the inverse to find the unknown values	$4x + 2y = 5$ $3x + 2y = 2$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 3 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

Concept	Definition/Explanation	Example
Unique Solution	If $X = A^{-1}B$, the inverse of A must exist for there to be a unique solution. For a unique solution $\text{Det}(A) \neq 0$	$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 3 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ This will have a unique solution as the $\text{Det}(A) = 2 \neq 0$
No Unique Solution	If $X = A^{-1}B$, the inverse of A must not exist for there not to be a unique solution. For no unique solution $\text{Det}(A) = 0$	$\begin{bmatrix} 3 & 4.5 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \end{bmatrix}$ This will have no unique solution as $\text{Det}(A) = 3 \times 3 - 2 \times 4.5 = 0$
Communication Matrix	A communication matrix gives the one step communication between participants.	 $C = \begin{matrix} & \text{To} \\ \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$
Two Step Communication Matrix	C^2 gives the two step communication matrix.	$C^2 = \begin{matrix} & \text{To} \\ \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 3 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{matrix}$
Total Communication Matrix	Is given by $T = C + C^2$ Where C is the one step communication matrix And C^2 is the two step communication matrix	$C + C^2 = \begin{matrix} & \text{To} \\ \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 2 & 2 & 2 & 1 \\ 2 & 2 & 2 & 1 \\ 2 & 2 & 3 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{matrix}$
Dominance Matrix	Is used to find the winner. Works in a similar way to communication matrices only in one direction.	 $D = \begin{matrix} & \text{Loser} \\ \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$
Total Dominance Matrix	Is given by $T = D + D^2$ Where D is the one step dominance matrix And D^2 is the two step dominance matrix To rank all the players: sum up the rows	$D + D^2 = \begin{matrix} & \text{Loser} \\ \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 2 & 1 & 2 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$
Transition Matrix	Shows the transition from one state to the next. Remember all columns must sum to 1.	 $T = \begin{matrix} & \text{Start} \\ \begin{matrix} X & Y & Z \end{matrix} \\ \begin{matrix} X \\ Y \\ Z \end{matrix} & \begin{bmatrix} 0.4 & 0.1 & 0.25 \\ 0.5 & 0.8 & 0.6 \\ 0.1 & 0.1 & 0.15 \end{bmatrix} \end{matrix}$
Recurrence Rule using a transition matrix	Since Transitions occur recursively, it can be modelled by a recurrence rule: $S_{n+1} = T \times S_n$	$T = \begin{bmatrix} 0.4 & 0.1 & 0.25 \\ 0.5 & 0.8 & 0.6 \\ 0.1 & 0.1 & 0.15 \end{bmatrix} \quad S_0 = \begin{bmatrix} 10 \\ 90 \\ 20 \end{bmatrix}$ $S_1 = \begin{bmatrix} 0.4 & 0.1 & 0.25 \\ 0.5 & 0.8 & 0.6 \\ 0.1 & 0.1 & 0.15 \end{bmatrix} \times \begin{bmatrix} 10 \\ 90 \\ 20 \end{bmatrix} = \begin{bmatrix} 18 \\ 89 \\ 13 \end{bmatrix}$
Rule after n transitions	Each transition requires the previous state to be pre-multiplied by the transition matrix. Therefore: $S_n = T^n \times S_0$	$S_2 = \begin{bmatrix} 0.4 & 0.1 & 0.25 \\ 0.5 & 0.8 & 0.6 \\ 0.1 & 0.1 & 0.15 \end{bmatrix}^2 \times \begin{bmatrix} 10 \\ 90 \\ 20 \end{bmatrix} = \begin{bmatrix} 19 \\ 88 \\ 13 \end{bmatrix}$
Steady State	After several transitions a closed system will reach steady state, where $S_{n+1} = S_n$	$T = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix} \quad S_0 = \begin{bmatrix} 30 \\ 60 \end{bmatrix}$ $S_{100} = T^{100} \times S_0 \approx \begin{bmatrix} 51 \\ 30 \end{bmatrix}$ $S_{101} = T^{101} \times S_0 \approx \begin{bmatrix} 51 \\ 30 \end{bmatrix}$
Non-closed system	Recurrence Relation given by $S_{n+1} = TS_n + B$ Must be completed recursively. Cannot use a rule. Does not reach steady state.	$T = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix} \quad S_0 = \begin{bmatrix} 30 \\ 60 \end{bmatrix} \quad B = \begin{bmatrix} 30 \\ 60 \end{bmatrix}$ $S_1 = T \times S_0 + B = \begin{bmatrix} 47 \\ 48 \end{bmatrix}$ $S_2 = T \times S_1 + B \approx \begin{bmatrix} 54 \\ 46 \end{bmatrix}$